ANCHOR CABLE DYNAMICS

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THESIS

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by

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June 1972

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
June 1972

ABSTRACT

A two dimensional model of an anchor cable subjected to a uniform current, fixed at the sea bed and moving with a specified motion at the surface is developed.

A steady state solution is developed and used as an initial condition for the numerical solution of the dynamic equations. The method of characteristics is used to carry out the numerical integration.

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TABLE OF SYMBOLS

$\tilde{A}$ = cross sectional area (in^2)

A_n' = cable acceleration in the normal direction (ft/sec^2)

A_s' = cable acceleration in the tangential direction (ft/sec^2)

C_d = drag coefficient for a cylinder

d = diameter of cable (in)

D' = drag force per unit length of cable (lbs/ft)

$\bar{Y}_{so}'$ = depth measured from the mean water line (ft)

$\tilde{E}$ = modulus of elasticity (lbs/in^2)

K = cable spring constant (lbs)

L = length of cable (ft)

M = mass per unit length of cable ($lb\text{-}sec^2/ft^2$)

M' = virtual mass per unit length of cable ($lb\text{-}sec^2/ft^2$)

ρ = density of seawater ($lb\text{-}sec^2/ft^4$)

Ψ = angle between the horizontal and a tangent to the cable at a point on the cable

S' = length along the cable measured from the fixed point on the sea bed (ft)

T' = tension (lbs)

t_0 = time scale (period for one cycle) (sec)

T_o = steady state tension of the cable at the surface (lbs)

t' = time (sec)

V_o = velocity scale (velocity amplitude at the surface) (ft/sec)

V' = current velocity (ft/sec)

V_s' = tangential cable velocity (ft/sec)

V_n' = normal cable velocity (ft/sec)

W = wieght per unit length of cable (lbs/ft)

Y_s' = vertical distance between the fixed point and the top of the cable (ft)

TABLE OF DIMENSIONLESS PARAMETERS

$V_n = V_n'/V_o$

$V_s = V_s'/V_o$

$T = T'/T_o$

$A = gt_o/V_o$

$B = T_o t_o/LMV_o$

$C = t_o W/V_o M'$

$D = V_o \rho C_d d t_o/2M'$

$E = V/V_o$

$F = L/V_o t_o$

$R = K/WL$

$C_s = (RA/F)^{\frac{1}{2}}$

$C_n = (BCT/AF)^{\frac{1}{2}}$

$Y_{so} = Y_{so}'/L$

$Y_s = Y_s'/L$

ACKNOWLEDGEMENT

The author would like to offer thanks to Dr. C. J. Garrison whose ideas and guidance provided the inspiration for this thesis; to Professor R. E. Newton for helpful review of the manuscript, to the faculty of the Mechanical Engineering Department, and to Cay for her patience.

I. INTRODUCTION

The study of anchor cable dynamics in recent years has been of some importance to those interested in the motion of vessels in seaways. The mooring of buoys for navigation or the mooring of large tanks to hold oil or other sea-transported goods are further examples of the importance of this study. The motion of ships riding at anchor in limited areas would be of interest, for example, to shipping companies that service small ports.

In a survey [Ref. 1] done by Casarella and Parsons, it was mentioned that the first experimental work on mooring cables was probably done in 1917 by Relf and Powell. Since then several people have worked on the mathematical model of this problem.

Through the years the model has changed to some extent. Some of the extensions considered have been the inclusion of tangential drag on the cable, the effect of surface waves, the effect of current, the effect of added mass, and also the effects of cross currents in the three dimensional models [Ref. 1]. Most of the studies done have been carried out for the static configuration; however, dynamic studies with arbitrary motion prescribed at the surface end of the cable have also been carried out.

With the advent of fast computers, time required to get numerical solutions has been greatly reduced; however, computer time is still considerable as will be shown in this analysis.

Wilson [Ref. 2] arrived at a model very similar to the one used in this thesis. Wilson considered the effect of surface waves as well as tangential drag on the cable; however, the computer program used to solve the problem was proprietary to the company which supported his research and was not published.

For the present analysis, the drag normal to the anchor cable only is considered and the effects of waves are neglected. The assumption of zero tangential drag has been included in the general approach to the problem and seems to be valid for reasonable currents and cable configurations. Surface waves are also neglected in this analysis as far as drag is concerned. The current applied is steady, uniform and in the horizontal plane.

In the dynamic study, the method of characteristics is used to solve the equations of motion using a digital computer.

II. THEORY

The problem under consideration is depicted in Figure 1. A cable of diameter d and weight W pounds per foot is anchored on the sea floor and is suspended from a point on the surface as indicated in Figure 1. A uniform current of magnitude V' is directed in the positive X-direction and the motion of the upper end of the cable may be specified in some suitable manner.

Writing Newton's Law for an elemental length of cable as depicted in the insert of Figure 1, the following dynamic equation is obtained:

$$T'\Delta\Psi - W\Delta s' \cos\Psi - D'\Delta s' = M'A_n'\Delta s' \qquad (1)$$

where A_n' is the normal acceleration.

Dividing equation (1) by $\Delta s'$ and taking the limit as $\Delta s' \to 0$ yields:

$$T' \frac{\partial \Psi}{\partial s'} - W\cos\Psi - D' = M'A_n' \qquad (2)$$

The term M' represents the virtual mass of the cable per unit length and D' is defined as the cable drag per unit of length. T' denotes the tension in the cable and the angle Ψ denotes the angle between the tangent at a point on the cable and the horizontal. In summing the forces in the tangential direction the added mass is taken as zero and as a result the

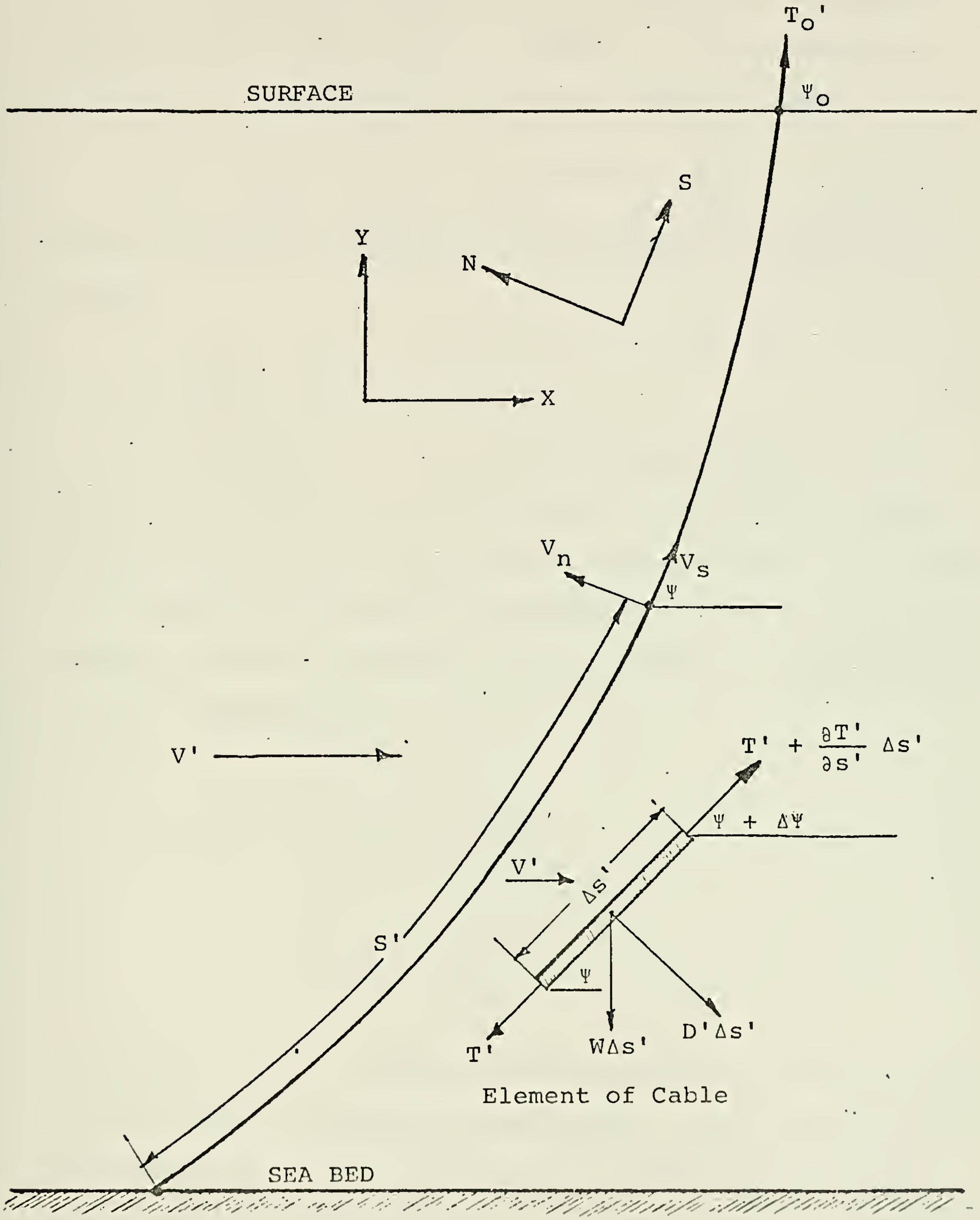


FIGURE 1 DEFINITION SKETCH

actual mass is simply the cable mass. Also, the tangential drag is omitted as stated in the introduction. From the forces shown in the insert of Figure 1, the equation of motion for the tangential direction takes the form:

$$-T' + (T' + \frac{\partial T'}{\partial s'} \Delta s')\cos \Delta\Psi - W\Delta s'\sin(\Psi + \frac{\Delta\Psi}{2}) = MA_s'\Delta s' \quad (3)$$

Again, dividing through by $\Delta s'$ and taking the limit as $\Delta s' \to 0$ yields:

$$\frac{\partial T'}{\partial s'} - W \sin \Psi = MA_s' \quad (4)$$

It is necessary also to develop expressions for the normal and tangential accelerations in terms of the normal and tangential velocities. Referring to Figure 2, the normal and tangential acceleration components as well as the corresponding velocity components can be written in terms of the x and y components as

$$A_s' = \ddot{X} \cos \Psi + \ddot{Y} \sin \Psi \quad (5)$$

$$A_n' = \ddot{Y} \cos \Psi - \ddot{X} \sin \Psi \quad (6)$$

$$V_s' = \dot{Y} \sin \Psi + \dot{X} \cos \Psi \quad (7)$$

$$V_n' = \dot{Y} \cos \Psi - \dot{X} \sin \Psi \quad (8)$$

in which $\dot{X}$, $\dot{Y}$, $\ddot{X}$, $\ddot{Y}$, denote the velocities and accelerations in the x and y directions, respectively. Taking the partial derivatives of equations (7) and (8) with respect to time and using the definitions of V_n' and V_s' as given in equations (7) and (8) it is possible to show that the normal and tangential accelerations may be written as:

$$A_n' = \frac{\partial V_n'}{\partial t'} + V_s' \frac{\partial \Psi}{\partial t'} \tag{9}$$

$$A_s' = \frac{\partial V_s'}{\partial t'} - V_n' \frac{\partial \Psi}{\partial t'} \tag{10}$$

The primes indicate that these terms are dimensional terms. Dimensionless forms of these parameters will be derived later.

Substituting equations (9) and (10) for the acceleration into equations (2) and (4), the following expressions for the two equations of motion are obtained:

$$T' \frac{\partial \Psi}{\partial s'} - W \cos \Psi - D' = M' \left(\frac{\partial V_n'}{\partial t'} + V_s' \frac{\partial \Psi}{\partial t'}\right) \tag{11}$$

$$\frac{\partial T'}{\partial s'} - W \sin \Psi = M \left(\frac{\partial V_s'}{\partial t'} - V_n' \frac{\partial \Psi}{\partial t'}\right) \tag{12}$$

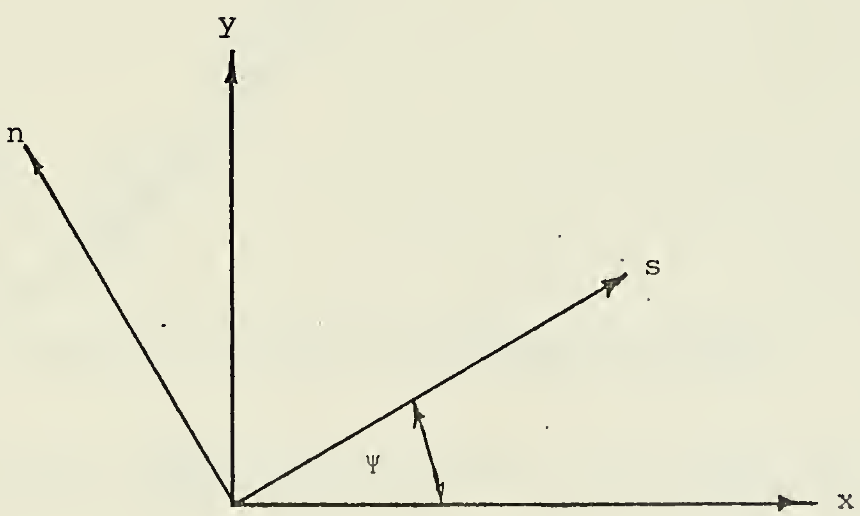


FIGURE 2 Coordinate Systems

From the dynamic equations, (11) and (12), it can be seen that there are four dependent variables, V_n', V_s', T', Ψ, all of which are functions of s' and t'. In order to solve the equations of motion two other equations are necessary. These are the two kinematic relations whose derivations follow.

With the assumption of a linear relationship between the stress and strain and referring to Fig. 3, it is possible to write the following relationship between the rate of increase of tension and the rate of strain,

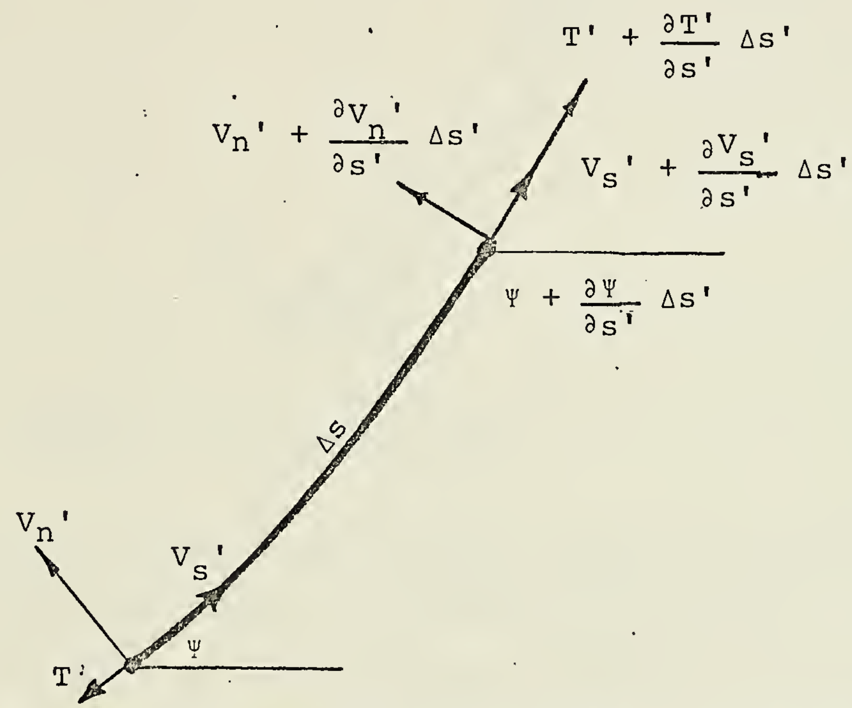


FIGURE 3 Tangential Forces on an Element of Cable

$$\frac{\Delta T'}{\Delta t'} = \frac{K}{\Delta s'} \left[\left(V_s' + \frac{\partial V_s'}{\partial s'} \Delta s'\right) \cos \Delta\Psi - V_s' - \left(V_n' + \frac{\partial V_n'}{\partial s'} \Delta s'\right) \sin \Delta\Psi \right] \quad (13)$$

which, upon taking the limit as $\Delta s' \to 0$, becomes:

$$\frac{1}{K} \frac{\partial T'}{\partial t'} = \frac{\partial V_s'}{\partial s'} - V_n' \frac{\partial \Psi}{\partial s'} \quad (14)$$

Equation (14) states that the rate of change of tension equals the "spring constant" times the rate of stretching. In the case of a solid cable of uniform cross section, K is simply $\tilde{A}\tilde{E}$ where $\tilde{A}$ is the cross sectional area and $\tilde{E}$ denotes the modulus of elasticity. In the case of, for example, a chain or stranded cable the value of K would become somewhat more complex and would most likely be determined experimentally.

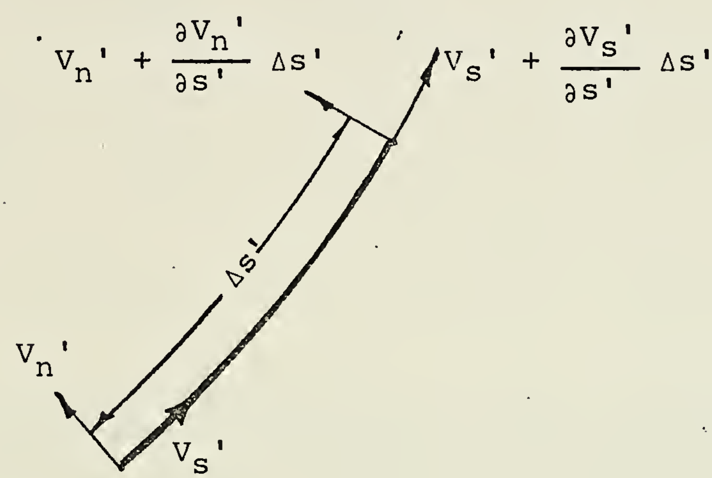


FIGURE 4 Cable Kinematics

A second kinematic relationship may also be introduced in the following manner. The angular velocity of a segment $\Delta s'$ of the cable may be specified in terms of the velocity of its end points. Referring to Figure 4, the angular velocity may be written as:

$$\frac{\partial \Psi}{\partial t'} = \frac{V_n' + \frac{\partial V_n'}{\partial s'} \Delta s' - V_n'}{\Delta s'} + V_s' \frac{\partial \Psi}{\partial s'} \frac{\Delta s'}{\Delta s'} \qquad (15)$$

which becomes upon taking the limit as $\Delta s' \to 0$,

$$\frac{\partial \Psi}{\partial t'} = \frac{\partial V_n'}{\partial s'} + V_s' \frac{\partial \Psi}{\partial s'} \qquad (16)$$

The four equations which have been developed, equations (11), (12), (14); (16), represent the complete system and are all that are required to determine the four dependent variables V_n', V_s', T', Ψ, in terms of the functions s' and t'. In summary, the basic system of equations is:

$$T' \frac{\partial \Psi}{\partial s'} - W \cos \Psi - D' = M' \left(\frac{\partial V_n'}{\partial t'} + V_s' \frac{\partial \Psi}{\partial t'}\right) \tag{17}$$

$$\frac{\partial T'}{\partial s'} - W \sin \Psi = M\left(\frac{\partial V_s'}{\partial t'} - V_n' \frac{\partial \Psi}{\partial t'}\right) \tag{18}$$

$$\frac{1}{K} \frac{\partial T'}{\partial t'} = \frac{\partial V_s'}{\partial s'} - V_n' \frac{\partial \Psi}{\partial s'} \tag{19}$$

$$\frac{\partial \Psi}{\partial t'} = \frac{\partial V_n'}{\partial s'} + V_s' \frac{\partial \Psi}{\partial s'} \tag{20}$$

The four equations which have now been specified equations (17) through (20), may be integrated numerically to obtain a solution. It is possible to carry out this numerical integration using a finite difference formulation in the s'-t' plane with a rectangular grid. However, it is also possible, and more convenient, to use the method of characteristics as outlined by, for example, Crandall [Ref. 3]. Using his suggested method it can be shown that the system is hyperbolic and equations (17) through (20) can be transformed into the form of first order differential equations valid along their respective characteristics.

According to Crandall, four other relationships in addition to equations (17) through (20) are needed to carry

out the integration by use of the method of characteristics. These relations are directional derivatives for the dependent variables V_n', V_s', T' and Ψ as defined by

$$DV_n' = \frac{\partial V_n'}{\partial s'} ds' + \frac{\partial V_n'}{\partial t'} dt' \tag{21}$$

$$DV_s' = \frac{\partial V_s'}{\partial s'} ds' + \frac{\partial V_s'}{\partial t'} dt' \tag{22}$$

$$D\Psi = \frac{\partial \Psi}{\partial s'} ds' + \frac{\partial \Psi}{\partial t'} dt' \tag{23}$$

$$DT' = \frac{\partial T'}{\partial s'} ds' + \frac{\partial T'}{\partial t'} dt' \tag{24}$$

Equations (21) through (24) denote incremental changes associated with small displacements ds' and dt' in the t'-s' plane. In order to determine the characteristics of the set of equations, equations (17) through (24) are set in a matrix form as follows:

$$\begin{bmatrix} 0 & M' & 0 & 0 & -T' & M'V_s' & 0 & 0 \\ 0 & 0 & 0 & M & 0 & -V_n'M & -1 & 0 \\ -1 & 0 & 0 & 0 & -V_s' & 1 & 0 & 0 \\ 0 & 0 & -K & 0 & V_n'K & 0 & 0 & 1 \\ ds' & dt' & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & ds' & dt' & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & ds' & dt' & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & ds' & dt' \end{bmatrix} \begin{bmatrix} \partial V_n'/\partial s' \\ \partial V_n'/\partial t' \\ \partial V_s'/\partial s' \\ \partial V_s'/\partial t' \\ \partial \Psi/\partial s' \\ \partial \Psi/\partial t' \\ \partial T'/\partial s' \\ \partial T'/\partial t' \end{bmatrix} = \begin{bmatrix} -W \cos \Psi - D' \\ -W \sin \Psi \\ 0 \\ 0 \\ DV_n' \\ DV_s' \\ D\Psi \\ DT' \end{bmatrix} \tag{25}$$

Expanding the determinant of the square matrix and equating it to zero the following result is obtained:

$$ds'^4 - \left(\frac{K}{M} + \frac{T'}{M'}\right) ds'^2 dt'^2 + \frac{T'K}{MM'} dt'^4 = 0 \tag{26}$$

Dividing by dt'^4 equation (26) becomes:

$$\left(\frac{ds'}{dt'}\right)^4 - \left(\frac{K}{M} + \frac{T'}{M'}\right)\left(\frac{ds'}{dt'}\right)^2 + \frac{T'K}{MM'} = 0 \tag{27}$$

Equation (27) is a quartic, the solution to which will produce four real characteristic directions.

Letting

$$C_s'^2 = \frac{K}{M} \tag{28}$$

$$C_n'^2 = \frac{T'}{M'} \tag{29}$$

and using these definitions equation (27) becomes:

$$\left(\frac{ds'}{dt'}\right)^4 - (C_s'^2 + C_n'^2)\left(\frac{ds'}{dt'}\right)^2 + C_s'^2 C_n'^2 = 0 \tag{30}$$

Solving now for ds'/dt' gives the four solutions:

$$\frac{ds'}{dt'} = \pm C_n' \tag{31}$$

$$\frac{ds'}{dt'} = \pm C_s' \tag{32}$$

These slopes define the four characteristics in the s'-t' plane associated with the system. The values C_s' and C_n' agree with those of Wilson [Ref. 2]. The fact that there are four real characteristics classifies the system as hyperbolic.

Having now determined the slopes of the characteristics, the next step is to determine the first order ordinary differential equations along each of these characteristics. To do this, the right hand side of the system matrix equation (Eq. 25) is substituted into any column of the square matrix and this determinant is expanded and set equal to zero. The first determinant is given in eq. (33).

$$\begin{vmatrix} 0 & M' & 0 & 0 & -T' & M'V_s' & 0 & -D'-W\cos\Psi \\ 0 & 0 & 0 & M & 0 & -MV_n' & -1 & -W\sin\Psi \\ -1 & 0 & 0 & 0 & -V_s' & 1 & 0 & 0 \\ 0 & 0 & -K & 0 & KV_n' & 0 & 0 & 0 \\ ds' & dt' & 0 & 0 & 0 & 0 & 0 & DV_n' \\ 0 & 0 & ds' & dt' & 0 & 0 & 0 & DV_s' \\ 0 & 0 & 0 & 0 & ds' & dt' & 0 & D\Psi \\ 0 & 0 & 0 & 0 & 0 & 0 & ds' & DT' \end{vmatrix} = 0 \qquad (33)$$

Expanding this determinant the following result is obtained:

$$\left(K - \frac{KC_n'^2}{C^2}\right)\frac{DV_s'}{Dt'} + \left(\frac{KV_n'C_n'^2}{C^2} - V_n'K\right)\frac{D\Psi}{Dt'} + \left(\frac{C_s'^2C_n'^2}{C^2} - C_s'^2\right)\frac{DT'}{Dt'}$$

$$= \left(\frac{C_n'^2C_s'^2}{C^2} - C_s'^2\right) W\sin\Psi \qquad (34)$$

where C_n', and C_s' are defined by equations (28) and (29) and $C = \frac{ds'}{dt'}$.

It may be noted that when $C = \pm C_n'$ both sides of the equation vanish. When C_s' is substituted for C the following two equations, valid along the $\pm C_s'$ characteristic, are obtained:

$$(+C_s'), \quad -\frac{D(V_s')}{Dt'} + V_n' \frac{D(\psi)}{Dt'} + \frac{1}{MC_s'} \frac{D(T')}{Dt'} = \frac{W \sin \Psi}{M} \tag{35}$$

$$(-C_s'), \quad \frac{D(V_s')}{Dt'} - V_n' \frac{D(\Psi)}{Dt'} + \frac{1}{MC_s'} \frac{D(T')}{Dt'} = \frac{-W \sin \Psi}{M} \tag{36}$$

Equations (35) and (36) are the ordinary differential equations valid along the $\pm C_s'$ characteristics; one for the positive slope and one for the negative slope. It is interesting to note here that the C_s' slopes are constant and therefore do not vary throughout the complete s'-t' plane.

Equations for the C_n' slopes are determined in a similar manner by substituting the right hand side of the matrix equation system (Eq. (25)) into another position in the square matrix; the fifth column will produce the desired result. After expanding, the following equation is obtained:

$$(C_s'^2 - C^2) \frac{D(V_n')}{Dt'} + C(C^2 - C_s^2) \frac{D(\psi)}{Dt'} + V_s'(C_s'^2 - C^2) \frac{D(\psi)}{Dt'}$$

$$= (C_s'^2 - C^2)\left(\frac{-W \cos \Psi + D'}{M'}\right) \tag{37}$$

In this case, if C is replaced by $\pm C_s'$ the equation becomes trivial. However, if $\pm C_n'$ is substituted for C the following equations, valid along the second characteristics C_n', are obtained:

$$(+C_n) \qquad \frac{D(V_n')}{Dt'} + (V_s' - C_n') \frac{D\Psi}{Dt'} = -\left(\frac{W \cos \Psi + D'}{M'} \right) \qquad (38)$$

$$(-C_n) \qquad \frac{D(V_n')}{Dt'} + (V_s' + C_n') \frac{D\Psi}{Dt'} = -\left(\frac{W \cos \Psi + D'}{M'} \right) \qquad (39)$$

Equations (35), (36), (38), and (39) represent four ordinary first order differential equations which are valid along their respective characteristics. The total derivatives $\frac{D}{Dt'}$ are defined according to $\frac{D}{Dt'} = \frac{\partial}{\partial t'} \pm C \frac{\partial}{\partial s'}$, where $C = \frac{ds'}{dt'}$ denotes the slope of the characteristic in the s'-t' plane. Since these equations are first order equations they are easily integrated along the characteristic directions.

With a uniform current directed as shown in Figure 1, the drag per unit length acting normal to the cable may be written in terms of the fluid velocity relative to the cable in the form

$$D' = \tfrac{1}{2}\rho C_d d(V' \sin \Psi + V_n')\,|V' \sin \Psi + V_n'| \qquad (40)$$

where ρ denotes the fluid density, C_d denotes the drag coefficient, V' denotes the current magnitude and d denotes the cable diameter or frontal area per unit length. The absolute value is used in order to maintain the sign of the relative velocity.

A. DIMENSIONLESS PARAMETERS

The four characteristic equations (35), (36), (38), and (39) can now be put in a dimensionless form. Various terms need to be defined before the actual analysis is done.

The solution to the problem as depicted in Figure 1 depends on a number of parameters. That is, the periodic motion at the surface may be characterized by a period (t_o) and a velocity amplitude (V_o). Also, at steady state, the tension at the surface T_o as well as an angle between the horizontal and a tangent Ψ_o to the cable may be specified. The unstretched length (L) of the cable can be used to normalize all length scales and the velocity amplitude may be used to normalize the velocities. Applying these definitions to the characteristic equations the following dimensionless parameters may be defined:

$$t = \frac{t'}{t_o} \tag{41}$$

$$V_s = \frac{V_s'}{V_o} \tag{42}$$

$$V_n = \frac{V_n'}{V_o} \tag{43}$$

$$T = \frac{T'}{T_o} \tag{44}$$

$$V = \frac{V'}{V_o} \tag{45}$$

$$C_n = C_n' \left(\frac{t_o}{L}\right) \tag{46}$$

$$C_s = C_s' \left(\frac{t_o}{L}\right) \qquad (47)$$

$$A = \frac{g t_o}{V_o} \qquad (48)$$

$$B = \frac{T_o t_o}{L M V_o} \qquad (49)$$

$$C = \frac{t_o W}{V_o M'} \qquad (50)$$

$$D = \frac{V_o \rho C_d d t_o}{2M'} \qquad (51)$$

$$E = \frac{V}{V_o} \qquad (52)$$

$$F = \frac{L}{V_o t_o} \qquad (53)$$

$$R = \frac{K}{WL} \qquad (54)$$

$$C_s = \left(\frac{RA}{F}\right)^{\frac{1}{2}} \qquad (55)$$

$$C_n = \left(\frac{BCT}{AF}\right)^{\frac{1}{2}} \qquad (56)$$

Applying the dimensionless groupings (41) through (56) to equations (35), (36), (38), and (39), the characteristic equations can be written in the following dimensionless form:

$$(+C_s) \qquad -\frac{D(V_s)}{Dt} + V_n \frac{D(\Psi)}{Dt} + \frac{B}{C_s}\frac{D(T)}{Dt} = A \sin \Psi \qquad (57)$$

$$(-C_s) \qquad \frac{D(V_s)}{Dt} - V_n \frac{D(\Psi)}{Dt} + \frac{B}{C_s}\frac{D(T)}{Dt} = -A \sin \Psi \qquad (58)$$

$$(+C_n) \quad \frac{D(V_n)}{Dt} + (V_s - FC_n)\frac{D\Psi}{Dt} = -C\cos\Psi - D[E\sin\Psi + V_n]|E\sin\Psi + V_n| \tag{59}$$

$$(-C_n) \quad \frac{D(V_n)}{Dt} + (V_s + FC_n)\frac{D\Psi}{Dt} = -C\cos\Psi - D[E\sin\Psi + V_n]|E\sin\Psi + V_n| \tag{60}$$

Equations (57) through (60) may now be solved numerically on the digital computer to obtain the dynamic solution. However, in order to proceed with the solution it is first necessary to determine an initial condition as well as appropriate boundary conditions on the surface and ocean bottom.

B. STEADY STATE SOLUTION

The steady state solution may be obtained by setting V_n, V_s and $\frac{\partial}{\partial t}$ equal to zero in equations (57) through (60). Accordingly, the following steady state form of equations (57) and (58) is obtained:

$$B\frac{dT}{ds} = A\sin\Psi \tag{61}$$

which is equivalent to equation (12) if the parameter definitions are substituted. Equations (59) and (60) become:

$$FC_n^2\frac{d\Psi}{ds} = C\cos\Psi + DE^2\sin^2\Psi \tag{62}$$

Equation (62) is equivalent to the equation of motion for the normal direction, equation (11). Since the characteristic equations agree with the equations of motion as far as steady state is concerned, equations (59) and (60) will be

the governing equations for the steady state solution. Also, the fact that both sets of equations agree is an indication that the characteristic equations were derived correctly.

Substituting equation (56) for C_n and dividing equation (61) by equation (62) provides the following expression:

$$\frac{dT}{T} = \frac{\sin \Psi \; d\Psi}{\cos \Psi + \frac{DE^2}{C} \sin^2 \Psi} \tag{63}$$

This equation can be integrated analytically between the limits of T to T_o and Ψ to Ψ_o. Upon integration the following result is obtained:

$$T^{-\left(1+\frac{4D^2E^2}{C^2}\right)^{\frac{1}{2}}} = \frac{\left(\frac{2DE^2}{C} \cos \Psi_o - 1 - \sqrt{1+\frac{4D^2E^4}{C^2}}\right) \left(\frac{2DE^2}{C} \cos \Psi - 1 + \sqrt{1+\frac{4D^2E^4}{C^2}}\right)}{\left(\frac{2DE^2}{C} \cos \Psi_o - 1 + \sqrt{1+\frac{4D^2E^4}{C^2}}\right) \left(\frac{2DE^2}{C} \cos \Psi - 1 - \sqrt{1+\frac{4D^2E^4}{C^2}}\right)} \tag{64}$$

If there should be zero current then equation (63) would reduce considerably since the parameter E would drop out. An analysis is done in Appendix B to compare equation (64) without the current term to the solution obtained directly from equation (63) setting E equal to zero.

Once T is known from equation (64) then the following expression can be used to get s:

$$ds = \frac{BCT \; d\Psi}{AC \cos \Psi + ADE^2 \sin^2 \Psi} \tag{65}$$

Equation (65) can not be analytically evaluated, and therefore it was integrated numerically on the digital computer.

C. BOUNDARY CONDITIONS

With the steady state solution used for the initial values only boundary conditions at the surface and sea bed need be specified. At the sea bed the normal and tangential velocities will be zero since the cable is anchored. With these two relations plus equations (58) and (60), the four dependent variables can be determined along the sea bed.

At the surface, equations (57) and (59) can be used; however, two other equations need to be specified such that the four dependent variables can be calculated. These two relations can be specified in several ways. For example, the surface velocities may be written as:

$$V_s = V_y \sin \Psi + V_x \cos \Psi \tag{66}$$

$$V_n = V_y \cos \Psi - V_x \sin \Psi \tag{67}$$

where V_x and V_y may be specified functions of time.

The two relationships required at the free surface may also take the form of the two-dimensional equations of motion for a floating vessel. In this case tension would be related to the position, velocity and acceleration of the floating object.

III. NUMERICAL PROCEDURE

Equations (57) through (60) represent four ordinary first order differential equations valid along their respective characteristics. In order to carry out the numerical integration, these equations are placed in the following difference form:

$$-V_{s5}+V_{n1}\Psi_5+\left(\frac{B}{C_s}\right)T_5 = A\sin\Psi_1\, Dt-V_{s1}+V_{n1}\Psi_1+\left(\frac{B}{C_s}\right)T_1 \tag{68}$$

$$V_{n5}+(V_{s2}-FC_{n2})\Psi_5 = (-C\cos\Psi_2-D[E\sin\Psi_2+V_{n2}]|E\sin\Psi_2+V_{n2}|)Dt +V_{n2}+(V_{s2}-FC_{n2})\Psi_2 \tag{69}$$

$$V_{n5}+(V_{s3}+FC_{n3})\Psi_5 = (-C\cos\Psi_3-D[E\sin\Psi_3+V_{n3}]|E\sin\Psi_3+V_{n3}|)Dt +V_{n3}+(V_{s3}+FC_{n3})\Psi_3 \tag{70}$$

$$V_{s5}-V_{n4}\Psi_5+\left(\frac{B}{C_s}\right)T_5 = -A\sin\Psi_4\, Dt+V_{s4}-V_{n4}\Psi_4+\left(\frac{B}{C_s}\right)T_4 \tag{71}$$

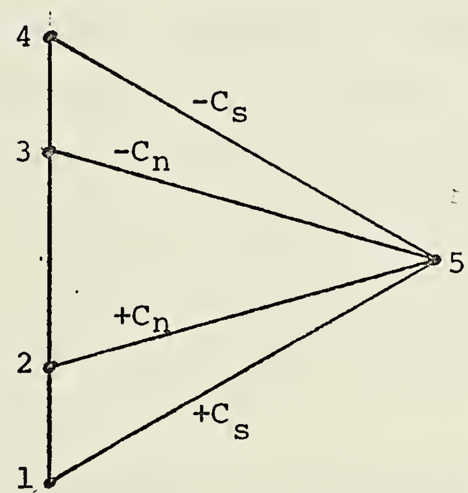


Figure 5 System of Characteristics

The subscripts 1,2,3,4 denote the initial point where it is supposed that all quantities V_s, V_n, T, and Ψ are known. In order to "march out" and determine new values at the new location denoted by the subscript 5, the four linear equations (68) through (71) are solved simultaneously. Letting the right hand sides of equations (68) through (71) be equal to AA, CC, DD, and BB, respectively, the resulting expressions for the dependent variables at the new location 5 are:

$$\Psi_5 = (CC - DD/((V_{s2} - V_{s3}) - F(C_{n2} + C_{n3})) \tag{72}$$

$$T_5 = (AA + BB + \Psi_5(V_{n4} - V_{n1}))/(\frac{2B}{C_s}) \tag{73}$$

$$V_{s5} = BB + \check{V}_{n4}\Psi_5 - (\frac{B}{C_s})T_5 \tag{74}$$

$$V_{n5} = CC - (V_{s2} - FC_{n2})\Psi_5 \tag{75}$$

The basic method of calculation indicated in equations (72) through (75) was used to determine values of the dependent variables V_s, V_n, T, and Ψ throughout the t-s plane. The numerical scheme utilized consisted of first dividing the s axis into equal subdivisions. The solution for V_s, V_n, T, and Ψ at t=0, or initial condition, was obtained from the steady state solution of equations (64) and (65). The steady state solution provided values for V_s, V_n, T, and Ψ at all of the nodal points along the S axis. The layout in the t-s plane is shown in Figure 6.

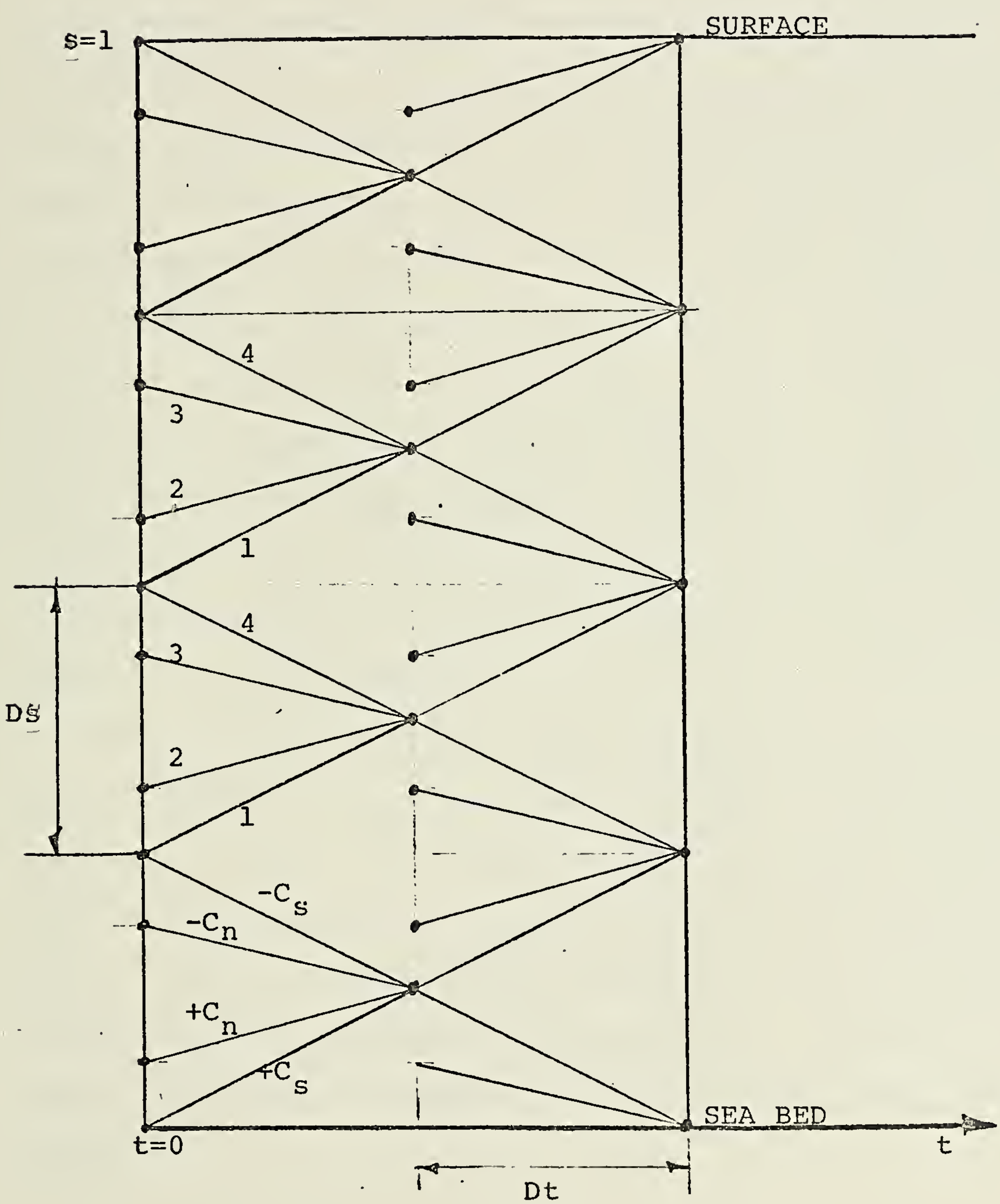
Using equations (72) through (75) a new set of values were then determined at the next row of points located at

time $Dt = Ds/C_s$ from the s axis. Once all the values of the four dependent variables were determined at this value of time, the solution was marched out another time step, Dt, so that values of the dependent variables were eventually obtained for a large area on the t-s plane.

It may be noted in Figures 5 and 6 that the $\pm C_n$ characteristics do not originate from a nodal point where values of the dependent variables are calculated. In the numerical procedure the method of calculation involved a linear interpolation between points 1 and 4 in Figure 5 in order to evaluate the dependent variables at points 2 and 3.

As indicated previously, the steady state solution was utilized to determine the initial values of the four dependent variables along the s axis. On the t axis, corresponding to the ocean floor, it is also necessary to apply some kind of boundary condition. In the example worked out, this boundary condition was taken as $V_n = V_s = 0$ which corresponds to a fixed anchor point.

FIGURE 6 Layout of Characteristics



IV. DISCUSSION OF RESULTS

In order to work a numerical example, it is necessary to specify values for the parameters appearing in equations (41) through (56). For the example worked, a steel cable of uniform cross section with a diameter (d) of 2 inches and a modulus of elasticity ($\tilde{E}$) of 30×10^6 psi was selected. The density of the steel was 480 lbs/ft^3 and the cable was 1000 feet long. The period (t_o) of the motion at the surface was 10 seconds and the velocity amplitude V_o was 3.0 feet/second. The density of seawater was taken to be 2.0 lbs-sec^2/ft^4. The virtual mass as described by $M'=M + C_m\rho \frac{\pi d^2}{4}$ where C_m denotes the added mass coefficient [Ref. 2], and ρ the density of seawater, was calculated to be 3.49. The added mass coefficient used in the above calculation was taken as 1.5. The drag coefficient C_d was taken as 1.5 which, according to Wilson [Ref. 2], is valid for Reynolds numbers less than 10^5. It is now possible to calculate values for the parameters in equations (48) through (56).

With an accurate steady state solution for tension having been calculated by equation (64), a check on the numerical integrations done in the characteristics method is possible. That is, it is possible to apply steady state boundary conditions at the free surface and recompute the steady state solution using the method of characteristic procedure. These results can then be compared with the original steady state solution.

In order to do this the steady state boundary condition at the surface was written in the following manner:

$$T \cos \Psi|_{s=1} = \cos \Psi_o \tag{76}$$

This equation states that the horizontal component of tension at the surface was held constant. The vertical component was written as:

$$T \sin \Psi|_{s=1} = \sin \Psi_o + C_o(Y_s - Y_{s_o}) \tag{77}$$

Equation (77) is a simplified expression representing the vertical component of tension as caused by a massless vessel floating on a still water surface in water of depth, Y_{s_o}. The dimensionless water depth Y_{s_o} is defined as $Y_{s_o} = \int_0^1 \sin \Psi \, ds|_{t=0}$ and likewise the vertical span of the cable at time t is defined as $y_s = \int_0^1 \sin \Psi \, ds|_t$. The product $C_o(y_s - y_{s_o})$ represents a buoyant force caused by a sinking of the vessel a distance of $(y_s - y_{s_o})$. The constant C_o is used to describe the interaction of the ship with the free surface. The "spring rate" C_o represents the slope of the buoyant force-sinkage curve and is accordingly defined as $C_o = (A_{wl})\gamma_{sw})L/T_o$ where A_{wl} is the water line area, γ_{sw} is the specific weight of seawater, and L is the length of the cable. Using equations (76) and (77) as the surface boundary condition, the method of characteristics was "marched out" several time steps in order to compare the results obtained from the method of characteristics procedure with the initial steady state results.

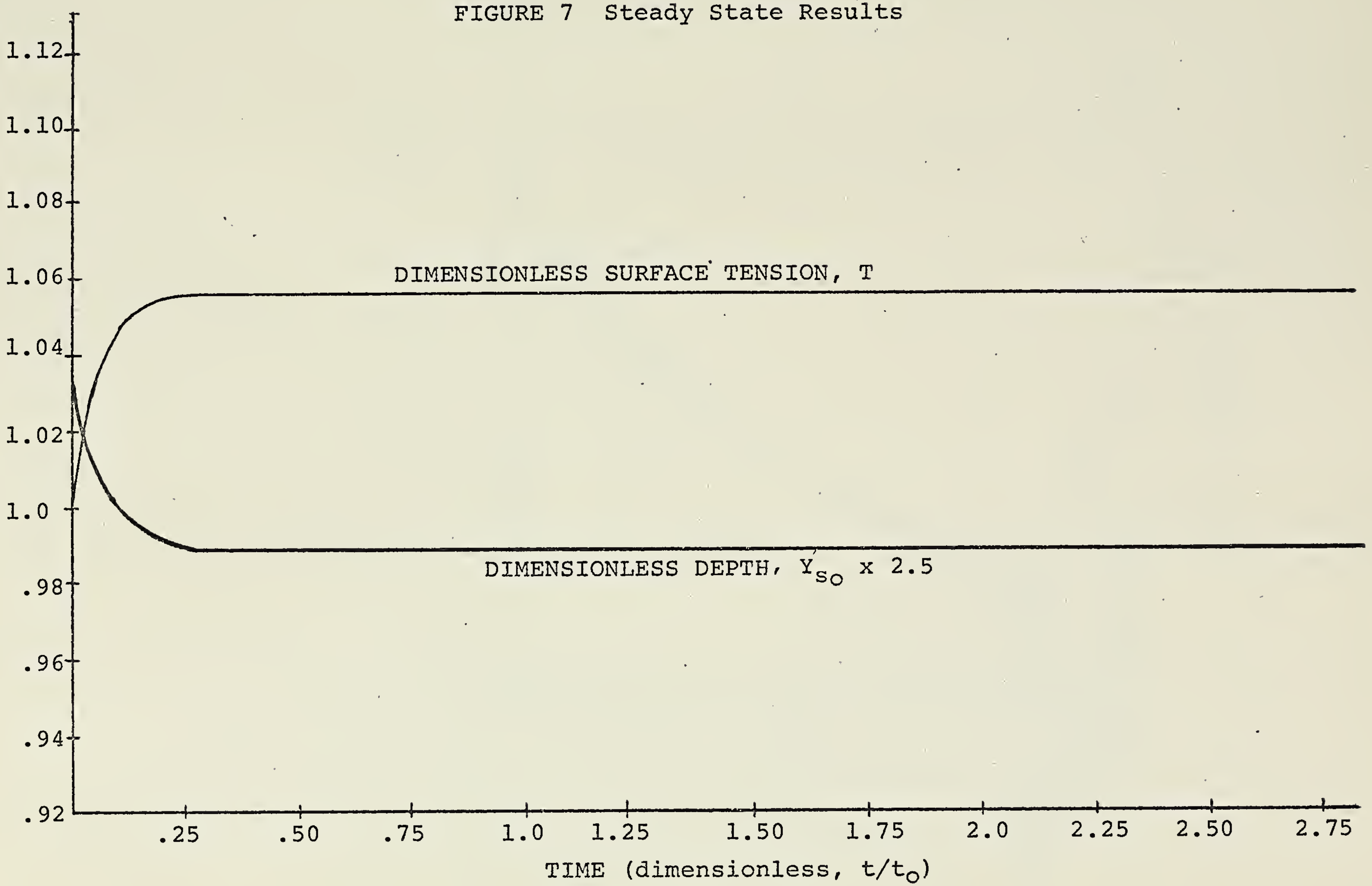
The results of the above described procedure are shown in Figure 7. The results corresponding to t=0 were calculated by use of equations (64) and (65). Using these initial values, the method of characteristics procedure was then used to continue this steady state solution for some time. It may be seen from Figure 7 that some drift in the results took place due to the numerical integration by the method of characteristics when compared to the initial solution. The figure shows that the vessel sank slightly as the tension increased. This change from the initial solution is not to be construed as inaccuracy, however. The new steady state solution simply represents a second possible steady state solution.

Since the steady state solution using the method of characteristics apparently provided good results, it was decided to add a dynamic boundary condition at the surface in order to induce a dynamic response of the cable. The boundary conditions at the surface, equations (76) and (77), were therefore modified slightly to include motion. Equation (76) was applied so that the horizontal component of tension was held constant. The vertical component of tension, however, was expressed in the following way:

$$T \sin \Psi \big|_{s=1} = \sin \Psi_o + C_o[(Y_s - Y_{s_o}) + A_m \sin 2\pi t] \qquad (78)$$

Figure 8 depicts the physical problem defined by the boundary condition equation (78). The equation describes the vertical component of tension as caused by a massless vessel floating in a wave of elevation $A_m \sin 2\pi t$ measured with

FIGURE 7 Steady State Results



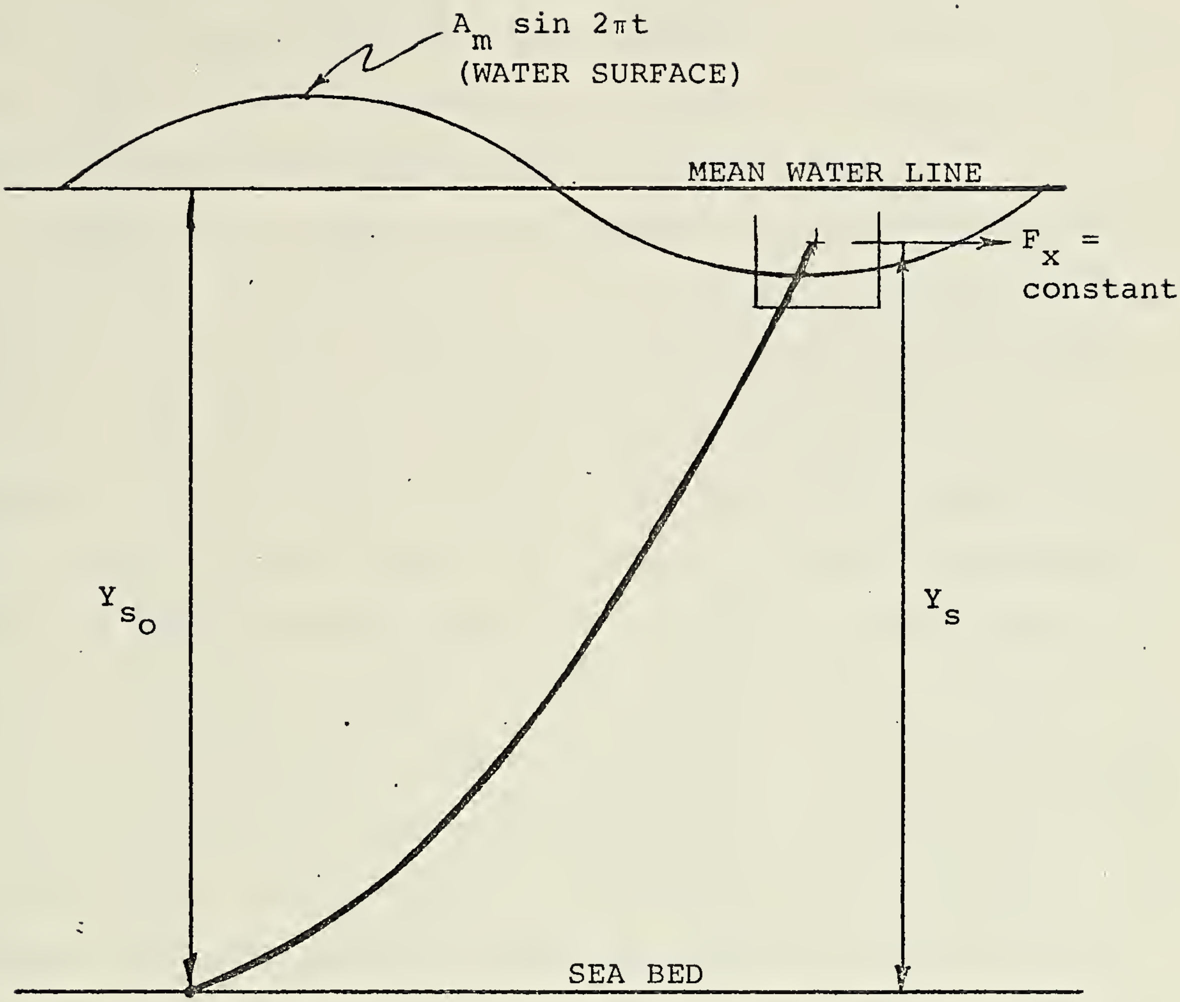


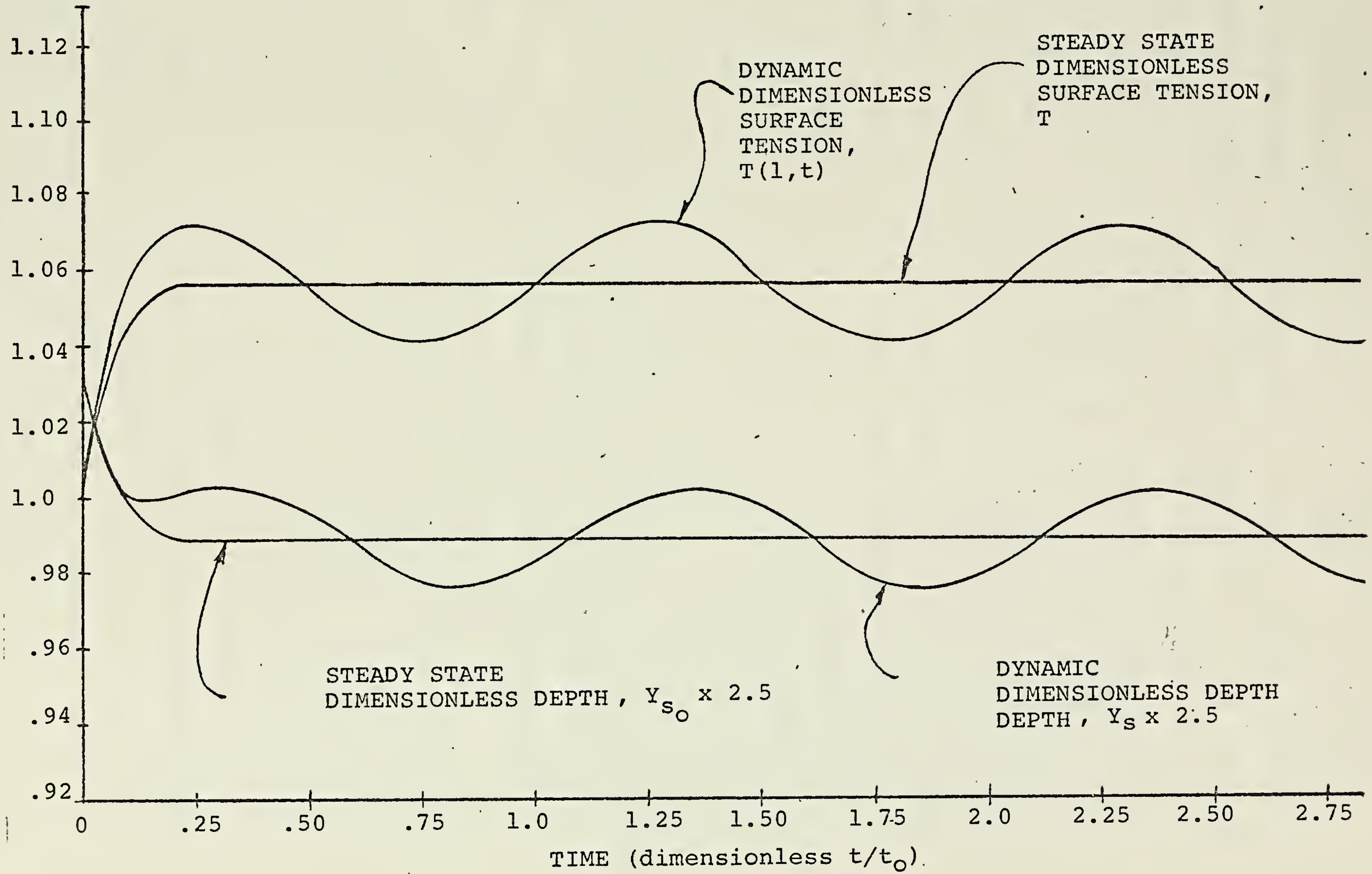
FIGURE 8 Dynamic System

respect to the mean water level. The term A_m represents the wave amplitude and is made dimensionless with the cable length. The terms within the square brackets represent the displacement of the vessel from its mean position with respect to the free water surface. As defined previously, C_o denotes the slope of the buoyant force versus sinkage curve.

Figure 9 is a graph of the dynamic response of the cable to the above described motion. On this graph are plotted the dimensionless tension and depth against a dimensionless time scale. Tension is made dimensionless with T_o which is defined as $T_o = WL/\sin \Psi_o$, $\Psi_o = 45°$ being the initial steady state angle at the surface. The depth is made dimensionless with the cable length, which is 1000 feet for the example worked. Finally, the time is made dimensionless with the period of the motion at the surface which is 10 seconds. The T_o calculated from the above expression is approximately 14,150 pounds. The results of the steady state analysis, Figure 7 is also shown on Figure 9. Figure 9 shows a sinusoidal response which is expected from the boundary condition driving the motion. Although the average tension at the surface should be near 1.0, it can be noticed that after about three cycles the tension appears to be oscillating around a mean value which has a 6 per cent shift. This new mean position is equal to the one calculated from the method of characteristics under steady state conditions.

Finally, it may be noted from Figure 9 that a phase shift exists between the position of the upper end of the cable and

FIGURE 9 Dynamic Tension and Depth



the tension. Specifically, the tension tends to lag the displacement somewhat. This lag is caused by the dynamics of the phenomena; a juxtaposition of static solutions would necessarily show the tension and displacement to be in phase.

The amount of computer time required to obtain numerical results depends on two factors, the number of subdivisions along the cable and the slope of the C_s characteristic. In general, the number of calculations required, and consequently, the computer time, is directly proportional to the square of the number of subdivisions along the s- axis or the cable length. The size of the time step advanced in each computation is related to the slope of the C_s characteristic according to $Dt = Ds/C_s$ where Ds is fixed by selection of the number of subdivisions of the cable and C_s is defined as $C_s = (K/M)^{1/2}$. Thus, in order to reduce the required computer time it is necessary to use the minimum number of subdivisions of the cable that will give accurate results and use small values of C_s. Unfortunately C_s is dependent on the cable physical properties and, accordingly, is fixed.

In view of the definition of C_s it is apparent that the time steps for very "stiff" cables with large elastic constants K will be small. However, for lighter cables with smaller elastic constants such as would correspond to nylon rope, the time steps will be larger and the calculation procedure more efficient. For the particular example worked, 30 subdivisions on the cable were used and it took approximately 30 minutes of computer time to compute 2.9 cycles corresponding to 29 seconds in real time.

The value of $C_s = (K/M)^{\frac{1}{2}}$ has the physical significance of being the speed of propagation of a stress wave along the cable while the second characteristic $C_n = (T'/M')^{\frac{1}{2}}$ has the physical significance of being the speed of propagation of a transverse wave along the cable. Accordingly, for all physically realistic cables the stress wave speed would normally be much larger than the transverse wave speed and consequently, C_s would be greater than C_n. In the case of steel cables, the stress wave speed is normally quite large making the time step small. Moreover, in the limiting case of the inextensible cable the calculation procedure breaks down because the time step becomes zero. However, in this case or in cases where the cable stretching can be disregarded, the problem actually becomes much simpler. That is, the system of four equations (57-60) are replaced by the two equations (59) and (60) which are valid along the $\pm C_n$ characteristics only. The $\pm C_s$ characteristics are lost completely and the system reduces to a much simpler two-characteristic system.

V. CONCLUSIONS

The mathematical model presented in this thesis provides a method for the solution of cable dynamic problems. Although a simplified boundary condition was employed on the surface for purposes of generating numerical results, the results obtained indicate that the method is valid and can be used in connection with any type of boundary condition on the surface.

Excessive computer time is the greatest problem to be overcome in producing numerical results and two methods for reducing these were discussed. In cases where stretching can be disregarded the computer time can be reduced by orders of magnitude by utilizing the two-characteristic model since the time step is in this case defined by $Dt = Ds/C_n$ and C_n is normally small compared to C_s. In cases where the cable is relatively stiff and yet long enough such that stretching cannot be disregarded, no solution to the problem of excessive computer time presents itself.

RECOMMENDATIONS

The following recommendations are advanced on the basis of experience gained during the course of the present study:

1. A surface boundary condition should be applied that contains all the terms involved in a surface vessel's motion including mass, added mass, damping and buoyant force.
2. The two-characteristic model should be developed and compared to the solutions using the four-characteristic method in order to evaluate the effect of stretching.
3. A study of the effect of cable subdivision size on accuracy should be carried out.
4. The model appearing in this thesis should be extended to include tangential drag, and surface wave drag on the cable and should be extended to the three dimensional case.

APPENDIX A

```
      DIMENSION T(200,2),PSI(200,2),S(100,2),AF(200,2)
      DIMENSION VN(30,2),VS(30,2)
      DIMENSION FN(50),DFN(50),ANG(51),AC(200,2)
C     THIS PROGRAM USES THE METHOD OF CHARACTERISTICS TO CALCULATE THE
C     STEADY STATE AND DYNAMIC RESPONSE OF A MOORING CABLE
C     PARAMETER VALUES FOLLOW
      A=107.33
      B=144.97
      C=9.973
      D=2.1428
      E=0.0
      F=33.3
      R=9000.
C     X DETERMINES THE NONDIMENSIONAL LENGTH OF THE CABLE
      X=1.0
C     TO IS THE PERIOD OF MOTION AT THE SURFACE
      TO=10.
C     GA IS THE SPECIFIC WEIGHT OF SEAWATER
      GA=60.
C     SL REPRESENTS THE VESSELS LENGTH
      SL=1.
C     BE REPRESENTS THE BEAM OF THE VESSEL
      BE=1.
C     SC IS THE CABLE LENGTH
      SC=1000.
      VXO=0.0
      VYO=0.0
C     W IS THE WEIGHT OF THE CABLE PER UNIT LENGTH
      W=10.
C     VO IS THE VELOCITY AMPLITUDE AT THE SURFACE
      VO=3.
C     PSIO IS THE STEADY STATE ANGLE AT THE SURFACE
      PSIO=.7860
C     AM IS THE WAVE AMPLITUDE AT THE SURFACE
      AM=.01
      EPS=.00001
C     EPS IS A CONVERGENCE CRITERIA FOR THE ITERATION FOR THE ANGLE AT
C     THE SURFACE,NI IS THE NUMBER OF ITERATIONS
      NI=50
C     NS IS THE NUMBER OF CABLE SUBDIVISIONS
      NS=30
C     NP IS THE NUMBER OF PERIODS DESIRED
```

```
      NP=3
C     NPTS IS A FINER SUBDIVISION OF THE CABLE USED IN SSTATE SUBROUTINE
      NPTS=100
      NPTS2=2*NPTS
      N=NS-1
      NN=NS-2
C     STEADY STATE IS CALLED FOR THE INITIAL CONDITION
      CALL SSTATE(A,B,C,E,PSIO,S,T,PSI,DEP,NPTS,NPTS2,AF,HORIZ,D,X,NS,VN
     1,VS,F,R,AC)
      PSIT=PSIO
C     TST IS THE STEADY STATE TENSION AT THE SURFACE
      TST=(W*SC)/(SIN(PSIT))
C     CO IS A BOUYANT CONSTANT REPRESENTING THE SHIPS INTERACTION WITH
C     THE FREE SURFACE
      CO=((SL)*(BE)*(SC)*(GA))/TST
C     CS,CN AND THEIR SUBSCRIPTS REPRESENT THE CHARACTERISTICS
      CS=SQRT((R*A)/F)
C     DELS IS THE ELEMENTAL LENGTH OF THE CABLE
      DELS=X/N
C     DT IS THE TIME INCREMENT
      DT=(DELS/2.)/CS
      NT=(NP/DT)
C     TPRI IS THE DIMENSIONLESS TIME
      TPRI=0.0
C     THIS IS THE OUTER LOOP WHICH CALCULATES TWO INCREMENTS OF TIME
C     EACH TIME THROUGH
      DO 100 M=1,NT,2
C     THE CALCULATION OF DEPTH AND HORIZONTAL LAY FOLLOW
      DEP=0.0
      HORIZ=0.0
      DO 398 I=1,N
      HORIZ=HORIZ+(COS((PSI(I,1)+PSI(I+1,1))/2))*(S(I+1,1)-S(I,1))
      DEP=DEP+(SIN((PSI(I,1)+PSI(I+1,1))/2))*(S(I+1,1)-S(I,1))
  398 CONTINUE
C     THIS LOOP IS FOR WRITING RESULTS AT THE DESIRED NUMBER OF TIMES
C     THROUGH THE LOOP
      DO 400 KK=1,NT,800
      IF(M.EQ.KK)GO TO 54
  400 CONTINUE
  395 GO TO 415
   54 WRITE(6,52)
   52 FORMAT('0','TPRI',10X,'B',10X,'DEP',10X,'HORIZ')
      WRITE(6,53)TPRI,B,DEP,HORIZ
   53 FORMAT(4F12.6)
      WRITE(6,55)
   55 FORMAT('0','PSI',10X,'T',10X,'S',10X,'VN',10X,'VS')
      WRITE(6,57)(PSI(I,1),T(I,1),S(I,1),VN(I,1),VS(I,1),I=1,NS)
   57 FORMAT(5F12.6)
```

```
  415 CONTINUE
      TPRI=TPRI+DT
      DO 20 J=1,N
      L=J+1
C     SOLVE SUBROUTINE SOLVE THE CHARACTERISTIC EQUATIONS FOR THE NEW
C     POINT IN THE NEXT TIME FRAME
C     NOTE THAT NO ITERATIONS WERE NECCESARY FOR THE BOUNDARY CONDITIONS
C     AT THE SURFACE OR SEA BED DURING THIS TIME INCREMENT
      CALL SOLVE(PSI(J,1),PSI(L,1),T(J,1),T(L,1),VN(J,1),VN(L,1),VS(J,1)
     1,VS(L,1),PSI(J,2),T(J,2),VN(J,2),VS(J,2),A,B,C,D,E,F,DT,CS,DELS)
   20 CONTINUE
C     THIS PROCESS IS SHIFTING THE POINTS BACK TO COLUMN ONE SO
C     THAT COLUMN TWO CAN BE USED FOR CALCULATIONS
      DO 30 K=1,N
      T(K,1)=T(K,2)
      PSI(K,1)=PSI(K,2)
      VN(K,1)=VN(K,2)
      VS(K,1)=VS(K,2)
   30 CONTINUE
      TPRI=TPRI+DT
C     THIS LOOP USES SOLVE TO CALCULATE THE SECOND TIME
C     INTERVAL.  THIS TIME BOUNDARY CONDITIONS NEED TO BE CALCULATED.
      DO 40 I=1,NS
   31 IF(I.EQ.1)GO TO 50
   32 IF(I.EQ.NS)GO TO 60
      J=I-1
      CALL SOLVE(PSI(J,1),PSI(I,1),T(J,1),T(I,1),VN(J,1),VN(I,1),VS(J,1)
     1,VS(I,1),PSI(I,2),T(I,2),VN(I,2),VS(I,2),A,B,C,D,E,F,DT,CS,DELS)
   33 GO TO 40
   50 CONTINUE
C     THIS IS THE BOTTOM BOUNDARY CONDITION
      VS(I,2)=0.0
      VN(I,2)=0.0
      PSI4=PSI(I,1)
      T4=T(I,1)
      VN4=VN(I,1)
      VS4=VS(I,1)
      T1=T(I+1,1)
      PSI1=PSI(I+1,1)
      VS1=VS(I+1,1)
      VN1=VN(I+1,1)
      CN1=SQRT((B*C*T1)/(A*F))
      CN4=SQRT((B*C*T4)/(A*F))
      DS4=(DELS/2.-DT*CN4)/(1.-(DT/DELS)*(CN1-CN4))
      T3=T4-(T1-T4)*(DS4/DELS)
      CN3=SQRT((B*C*T3)/(A*F))
      PSI3=PSI4-(PSI1-PSI4)*(DS4/DELS)
      VS3=VS4-(VS1-VS4)*(DS4/DELS)
```

```
      VN3=VN4-(VN1-VN4)*(DS4/DELS)
      BB=(-A*SIN(PSI4))*DT+VS4-(VN4)*PSI4+(B/CS)*T4
      DD=(-C*COS(PSI3)-D*(E*SIN(PSI3)+VN3)*(ABS(E*SIN(PSI3)+VN3)))*DT+VN
     13+(VS3+F*CN3)*PSI3
      PSI(I,2)=DD/(VS3+F*CN3)
      T(I,2)=(BB+VN4*(PSI(I,2)))/(B/CS)
   49 GO TO 40
   60 CONTINUE
      CN4=SQRT((B*C*(T(N,1)))/(A*F))
      CN1=SQRT((B*C*(T(NN,1)))/(A*F))
      DS1=((DELS/2.)-(CN4)*(DT))/(1.+(CN4-CN1)*(DT/DELS))
      T2=(T(N,1)-T(NN,1))*(DS1/DELS)+T(N,1)
      PSI2=(PSI(N,1)-PSI(NN,1))*(DS1/DELS)+PSI(N,1)
      VS2=(VS(N,1)-VS(NN,1))*(DS1/DELS)+VS(N,1)
      VN2=(VN(N,1)-VN(NN,1))*(DS1/DELS)+VN(N,1)
      CN2=SQRT((B*C*T2)/(A*F))
      CC=(-C*COS(PSI2)-D*(E*SIN(PSI2)+VN2)*(ABS(E*SIN(PSI2)+VN2)))*DT+VN
     12+(VS2-F*CN2)*PSI2
   74 IF(M.GT.1)GO TO 41
      ANG(1)=PSI2
      DO 70 K=1,NI
      FN(K)=(VS2-F*CN2)*ANG(K)-VXO*SIN(ANG(K))+VYO*COS(ANG(K))-CC
      DFN(K)=VS2-F*CN2-VXO*COS(ANG(K))-VYO*SIN(ANG(K))
      ANG(K+1)=ANG(K)-(FN(K))/(DFN(K))
      DIFF=ABS(ANG(K+1)-ANG(K))
   56 IF(DIFF-EPS)35,35,70
   35 PSI(NS,2)=ANG(K+1)
   37 GO TO 75
   70 CONTINUE
      WRITE(6,212)
  212 FORMAT('0','FAILED TO CONVERGE')
   75 CONTINUE
C     THIS IS THE  SURFACE BOUNDARY CONDITION THE FIRST TIME THROUGH
      VN(NS,2)=-VXO*SIN(PSI(NS,2))+VYO*COS(PSI(NS,2))
      VS(NS,2)=VYO*SIN(PSI(NS,2))+VXO*COS(PSI(NS,2))
      T1=T(N,1)
      PSI1=PSI(N,1)
      VS1=VS(N,1)
      VN1=VN(N,1)
      AA=(A*SIN(PSI1))*DT-VS1+(VN1)*PSI1+(B/CS)*T1
      PSIO=PSI(NS,2)
      T(NS,2)=(AA+VS(NS,2)-(VN1*PSIO))/(B/CS)
   40 CONTINUE
      DEP=0.0
      HORIZ=0.0
C     THIS LOOP CALCULATES THE DEPTH AND THE HORIZONTAL PROJECTION OF
C     THE CABLE ON THE SEA BED
      DO 600 I=1,N
```

```
      DEP=DEP+(SIN((PSI(I,2)+PSI(I+1,2))/2))*(S(I+1,1)-S(I,1))
      HORIZ=HORIZ+(COS((PSI(I,2)+PSI(I+1,2))/2))*(S(I+1,1)-S(I,1))
  600 CONTINUE
      IF(M.EQ.1)DEPO=DEP
      IF(M.EQ.1)HORIZO=HORIZ
   41 T1=T(N,1)
      PSI1=PSI(N,1)
      VS1=VS(N,1)
      VN1=VN(N,1)
      AA=(A*SIN(PSI1))*DT-VS1+(VN1)*PSI1+(B/CS)*T1
      TS=(W*SC)/(TST*SIN(PSIO))
      BC=TS*SIN(PSIO) + CO*((DEPO-DEP)+AM*SIN(6.283*TPRI))
C     THIS IS THE SURFACE BOUNDARY CONDITION AT ALL OTHER TIMES
      T(NS,2)=SQRT((TS*COS(PSIO))**2+(BC)**2)
      PSI(NS,2)=ARCOS((TS*COS(PSIO))/T(NS,2))
      VN(NS,2)=CC-(VS2-F*CN2)*PSI(NS,2)
      VS(NS,2)=VN1*PSI(NS,2)+(B/CS)*T(NS,2)-AA
      DO 95 K=1,NS
      T(K,1)=T(K,2)
      PSI(K,1)=PSI(K,2)
      VN(K,1)=VN(K,2)
      VS(K,1)=VS(K,2)
   95 CONTINUE
  100 CONTINUE
      WRITE(6,65)
   65 FORMAT('0','B',10X,'DEP',10X,'HORIZ')
      WRITE(6,67)B,DEP,HORIZ
   67 FORMAT('0',F8.4,4X,F8.4,6X,F8.4)
      WRITE(6,85)
   85 FORMAT('0','PSI',10X,'T',10X,'S',10X,'VN',10X,'VS')
      WRITE(6,87)(PSI(I,1),T(I,1),S(I,1),VN(I,1),VS(I,1),I=1,NS)
   87 FORMAT('0',F8.4,4X,F8.4,4X,F8.4,4X,F8.4,4X,F8.4)
      STOP
      END


      SUBROUTINE SSTATE(A,B,C,E,PSIO,S,T,PSI,DEP,NPTS,NPTS2,AF,HORIZ,D,
     1X,NS,VN,VS,F,R,AC)
      DIMENSION T(NPTS2,2),PSI(NPTS2,2),S(NPTS,2),AF(NPTS2,2)
      DIMENSION VN(NS,2),VS(NS,2),AC(NPTS2,2)
C     THIS PROGRAM CALCULATES THE STEADY STATE T,S,AND PSI WITH ZERO
C     TANGENTIAL DRAG
C     THIS INTEGRATION STARTS AT THE SEA BED AND GOES TO THE SURFACE
      L=NPTS*2-1
      M=NPTS-1
      N=NS-1
C     FIRST GENERATE THE ANGLE PSI(I)
      DPSI=PSIO/(2*(NPTS-1))
```

```
      IF(E.LE..05) GO TO 10
      DO 5 I=1,L
      PSI(I,2)=DPSI*(I-1)
C     CALCULATE TENSION VIA ANALYTICAL TENSION FORMULA
      Y=1./(COS(PSI(I,2))+((D*(E**2))*(SIN(PSI(I,2)))**2)/C)
      TA=(2.*D*(E**2)/C)*COS(PSIO)-1.-SQRT(1.+(4.*(D**2)*((E**2)**2))/(C
     1**2))
      TB=(2.*D*(E**2)/C)*COS(PSI(I,2))-1.+SQRT(1.+(4.*(D**2)*((E**2)**2
     1))/(C**2))
      TC=(2.*D*(E**2)/C)*COS(PSIO)-1.+SQRT(1.+(4.*(D**2)*((E**2)**2))/(C
     1**2))
      TD=(2.*D*(E**2)/C)*COS(PSI(I,2))-1.-SQRT(1.+(4.*(D**2)*((E**2)**2
     1))/(C**2))
      T(I,2)=((TA*TB)/(TC*TD))**(-1./(SQRT(1.+(4.*(D**2)*((E**2)**2))/(
     1C**2))))
C     GENERATE FUNCTION FOR NUMERICAL INTEGRATION TO GET S(I)
      AF(I,2)=Y*T(I,2)
    5 CONTINUE
      GO TO 17
C     THIS LOOP CALCULATES TENSION WITH ZERO OR CURRENT LESS THAN .05
C     KNOTS
   10 DO 15 J=1,L
      PSI(J,2)=DPSI*(J-1)
      T(J,2)=(COS(PSIO))/(COS(PSI(J,2)))
      AF(J,2)=(T(J,2))/(COS(PSI(J,2)))
   15 CONTINUE
C     SIMPSONS RULE
   17 SIMP1=0.0
      DO 20 K=1,M
      SIMP2=(AF(2*K-1,2)+4.*AF(2*K,2)+AF(2*K+1,2))*DPSI/3.
      SIMP1=SIMP1+SIMP2
   20 CONTINUE
      B=A/SIMP1
      IF(E.LE..05)GO TO 21
      DO 22 J=1,L
      Y=(B*C)/(A*C*COS(PSI(J,2))+(A*D)*(E**2)*(SIN(PSI(J,2)))**2)
      AC(J,2)=Y*(T(J,2))
   22 CONTINUE
      GO TO 41
   21 DO 23 I=1,L
      AC(I,2)=(T(I,2))*(B/(A*COS(PSI(I,2))))
   23 CONTINUE
   41 CONTINUE
C     USE SIMPSONS RULE TO GET S(I)
      S(1,2)=0.0
      SIMC1=0.0
      DO 24 K=1,M
      SIMC2=(AC(2*K-1,2)+4.*AC(2*K,2)+AC(2*K+1,2))*DPSI/3.
```

```
      SIMC1=SIMC1+SIMC2
      S(K+1,2)=SIMC1
   24 CONTINUE
      DO 30 I=1,NPTS
C     CHANGE PSI AND T BACK TO LARGER SUBDIVISION
C     PICK UP EVERY OTHER POINT
      PSI(I,2)=PSI(2*I-1,2)
      T(I,2)=T(2*I-1,2)
   30 CONTINUE
C     ADJUST S(I) SO THAT IT EQUALS 1.0 AT THE TOP
      DO 40 I=1,NPTS
      S(I,2)=S(I,2)/S(NPTS,2)
   40 CONTINUE
C     THIS LOOP CALCULATES THE DEPTH AND THE HORIZONTAL PROJECTION OF
C     THE CABLE ON THE SEA BED
      HORIZ=0.0
      DEP=0.0
      DO 51 I=1,M
      DEP=DEP+(SIN((PSI(I,2)+PSI(I+1,2))/2))*(S(I+1,2)-S(I,2))
      HORIZ=HORIZ+(COS((PSI(I,2)+PSI(I+1,2))/2))*(S(I+1,2)-S(I,2))
   51 CONTINUE
C     THIS PART OF THE PROGRAM MAKES EQUAL SPACING ALONG S AND
C     INTERPOLATES FOR T AND PSI FROM THE VALUES ABOVE
      DELS=X/N
      S(1,1)=0.0
      PSI(1,1)=PSI(1,2)
      T(1,1)=T(1,2)
      DO 55 I=1,N
      S(I+1,1)=DELS*(I)
   55 CONTINUE
      DO 90 I=1,N
      DO 70 J=1,M
      IF(S(I+1,1)-S(J+1,2)) 65,60,70
   60 T(I+1,1)=T(J+1,2)
      GO TO 90
   65 T(I+1,1)=T(J,2)+(T(J+1,2)-T(J,2))*((S(I+1,1)-S(J,2))/(S(J+1,2)-S
     1(J,2)))
      PSI(I+1,1)=PSI(J,2)+(PSI(J+1,2)-PSI(J,2))*((S(I+1,1)-S(J,2))/(S(
     1J+1,2)-S(J,2)))
      GO TO 90
   70 CONTINUE
   90 CONTINUE
      DO 100 K=1,NS
      VN(K,1)=0.00000
      VS(K,1)=0.00000
  100 CONTINUE
      JP=0
      DO 150 I=1,NS
```

```
      IF(S(I,1).GT.1.0) JP=JP+1
150   CONTINUE
      NP=NS-JP
      DO 200 I=1,NP
      PSI(NS-I+1,1)=PSI(NP-I+1,1)
      T(NS-I+1,1)=T(NP-I+1,1)
200   CONTINUE
      DO 300 I=1,JP
      T(I,1)=T(JP+1,1)
      PSI(I,1)=PSI(JP+1,1)
300   CONTINUE
      RETURN
      END


      SUBROUTINE SOLVE(PSI1,PSI4,T1,T4,VN1,VN4,VS1,VS4,PSI,T,VN,VS,A,B,
     1C,D,E,F,DT,CS,DELS)
      CN1=SQRT((B*C*T1)/(A*F))
      CN4=SQRT((B*C*T4)/(A*F))
      DS1=((DELS/2.)-DT*CN1)/((CN4-CN1)*(DT/DELS)+1.)
      DS4=((DELS/2.)-DT*CN4)/(1.-(CN4-CN1)*(DT/DELS))
      T2=(T4-T1)*(DS1/DELS)+T1
      T3=(T4-T1)*((DELS-DS4)/DELS)+T1
      CN2=SQRT((B*C*T2)/(A*F))
      CN3=SQRT((B*C*T3)/(A*F))
      PSI2=(PSI4-PSI1)*(DS1/DELS)+PSI1
      PSI3=(PSI4-PSI1)*((DELS-DS4)/DELS)+PSI1
      VN2=(VN4-VN1)*(DS1/DELS)+VN1
      VN3=(VN4-VN1)*((DELS-DS4)/DELS)+VN1
      VS2=(VS4-VS1)*(DS1/DELS)+VS1
      VS3=(VS4-VS1)*((DELS-DS4)/DELS)+VS1
      AA=(A*SIN(PSI1))*DT-VS1+(VN1)*PSI1+(B/CS)*T1
      BB=(-A*SIN(PSI4))*DT+VS4-(VN4)*PSI4+(B/CS)*T4
      CC=(-C*COS(PSI2)-D*(E*SIN(PSI2)+VN2)*(ABS(E*SIN(PSI2)+VN2)))*DT+VN
     12+(VS2-F*CN2)*PSI2
      DD=(-C*COS(PSI3)-D*(E*SIN(PSI3)+VN3)*(ABS(E*SIN(PSI3)+VN3)))*DT+VN
     13+(VS3+F*CN3)*PSI3
      PSI=(CC-DD)/((VS2-VS3)-F*(CN2+CN3))
      T=(AA+BB+PSI*(VN4-VN1))/((2.*B)/CS)
      VS=BB+(VN4)*PSI-(B/CS)*T
      VN=CC-(VS2-F*CN2)*PSI
      RETURN
      END
```

APPENDIX B

It is interesting to note that if the current is zero then equation (63) becomes:

$$\frac{dT}{T} = \frac{\sin \Psi \, d\Psi}{\cos \Psi} \tag{78}$$

This equation can be easily integrated into the following form:

$$T = \frac{\cos \Psi_o}{\cos \Psi} \tag{79}$$

By setting the current equal to zero in equation (64) it is not apparent that the result will be the same as equation (79). It is possible to show that equation (64) does reduce to equation (79) by the following method.

Letting $\qquad \varepsilon = \dfrac{4E^4D^2}{C^2}$.

and using a binomial expansion for $(1+\varepsilon)^{\frac{1}{2}}$ the following is obtained:

$$(1+\varepsilon)^{\frac{1}{2}} = 1 + \frac{1}{2}\varepsilon - \frac{1}{8}\varepsilon^2 + \frac{1}{16}\varepsilon^3 \tag{80}$$

Substituting equation (80) into the numerator of equation (64) produces the following numerator:

$$(-2 - \frac{1}{2}\varepsilon^{\frac{1}{2}}\cos \Psi_o - \frac{1}{2}\varepsilon^2 - \frac{1}{16}\varepsilon^3)(-\frac{1}{2}\varepsilon^{\frac{1}{2}}\cos \Psi + \frac{1}{2}\varepsilon - \frac{1}{8}\varepsilon^2 + \frac{1}{16}\varepsilon^3) \tag{81}$$

The coefficient of the $\varepsilon^{\frac{1}{2}}$ term will produce $\cos\Psi$ since the other terms will be of higher order. A similar analysis can be done in the denominator producing a coefficient of $\cos\Psi_o$.

Equation (64) now reduces to:

$$T = \frac{\cos\Psi_o}{\cos\Psi} \tag{82}$$

which is identical to equation (79).

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DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP
3. REPORT TITLE Anchor Cable Dynamics	
4. DESCRIPTIVE NOTES (Type of report and, inclusive dates) Master's Thesis; June 1972	
5. AUTHOR(S) (First name, middle initial, last name) Lance Cameron Horne	

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
June 1972	55	5

8a. CONTRACT OR GRANT NO. b. PROJECT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
c. d.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
10. DISTRIBUTION STATEMENT Approved for public release; distribution unlimited.	
11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

A two dimensional model of an anchor cable subjected to a uniform current, fixed at the sea bed and moving with a specified motion at the surface is developed.

A steady state solution is developed and used as an initial condition for the numerical solution of the dynamic equations. The method of characteristics is used to carry out the numerical integration.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6811

A-31408

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Anchor Cables						
Mooring Cables						
Cable Dynamics						
Cable Hydrodynamics						

Thesis
H8025 Horne 135269
c.1 Anchor cable dynamics.